Outline

1. The Stark effect
2. The standard model
3. The Impact Approximation

Spectral line broadening

- time dependent Schrödinger equation is

$$i\frac{\partial\Psi}{\partial t} = H\Psi$$

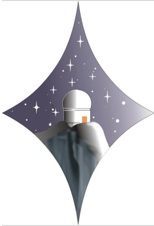
- If H is time independent. One solution is

$$f(t) = \psi_k e^{-iE_k t}$$

- ψ_k, E_k are an eigenvector and eigenvalue of H.
- power spectrum

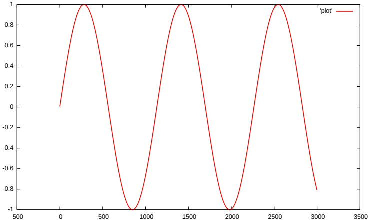
$$I(\omega) \propto |\int f(t)\exp(i\omega t)\,dt|^2$$

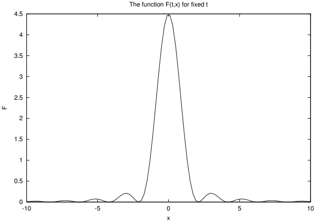
- proportional to square of the Fourier transform
- a delta function $\delta(\omega - \omega_0)$
- use angular frequencies so $E = \hbar\omega_0$
- infinitely narrow line – no interaction with outside world.



- wave train is now finite

$$I(\omega) \propto \left| \frac{\sin[\tau(\omega-\omega_0)/2]}{(\omega-\omega_0)/2} \right|^2$$





- Lorentz: distribution of mean time τ_c between collisions

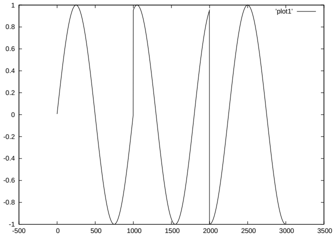
$$P(\tau)\,d\tau = \exp(-\tau/\tau_c)\,d\tau/\tau_c$$

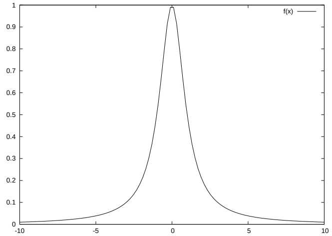
- spectrum is then

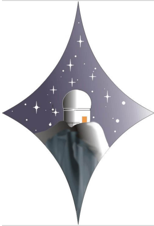
$$I(\omega) \propto \int_0^\infty \left| \frac{\sin[\tau(\omega-\omega_0)/2]}{(\omega-\omega_0)/2} \right|^2 \exp(-\tau/\tau_c)\,d\tau$$

or a Lorentz profile

$$I(\omega) \propto \frac{1}{(\omega-\omega_0)^2 + (1/\tau_c)^2}$$







Radiative lifetime

- lifetime due to radiative transitions is

$$\Gamma \equiv \tau_j^{-1} = \sum_i A_{ji}$$

A_{ji} are Einstein coefficients

- total width is sum of two terms, for upper and for lower
- profile is Lorentz
- Must add Doppler broadening $\Longrightarrow$ Voigt profile

$$H(a,v) = \frac{a}{\pi}\int_{-\infty}^{\infty} \frac{e^{-y^2}\,dy}{(v-y)^2 + a^2}$$

with $v \equiv \frac{(\nu-\nu_0)}{\Delta\nu_D}$, $y \equiv \frac{\Delta\nu}{\Delta\nu_D}$, $a \equiv \left(\frac{\gamma}{4\pi\Delta\nu_D}\right)$ and Doppler width $\Delta\nu_D = \left(\frac{2kT}{M}\right)^{\frac{1}{2}} \frac{\nu_0}{c}$

- is not correct in principle, radiator moves
- is a good approximation though

The Stark effect

- electric field gives potential

$$\Phi(\mathbf{r}) = E_z z$$

- so Hamiltonian is

$$H = -\sum_{i=1}^{N} \frac{p_i^2}{2\mu} + V + E_z \sum_{i=1}^{N} z_i .$$

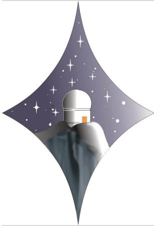
- Time-independent perturbation theory gives first-order energy correction

$$\Delta E_n^{(1)} = E_z \langle \Psi_n | \sum_{i=1}^{N} z_i | \Psi_n \rangle$$

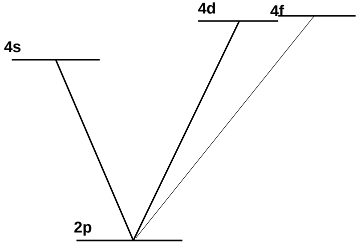
- z is odd, Ψ_n is either even or odd $\implies$ zero.
- second order term gives quadratic Stark

$$\Delta E_n^{(2)} = (E_z)^2 \sum_{m \neq n} \frac{|\langle \Psi_n | \sum_{i=1}^{N} z_i | \Psi_m \rangle|^2}{E_n - E_m}$$

- For hydrogen-like simple perturbation theory not applicable (energies degenerate). Linear effect.



- high n also linear effect.



- line-broadening and level dissolution linked

The standard model

- few radiators in a sea pertubers
- radiators do not interact with each other
- radiator has no effect on the perturbers (no back-reaction)
- can use "Classical path approximation"
- Note density cannot be too low.
- dipole interaction so transition probability is

$$\frac{4\omega_{if}}{3\hbar c^3}|\langle f|\mathbf{d}|i\rangle|^2.$$

- in far wings correct profile must join smoothly onto free-free opacity

- total power is weighted sum over all possible initial states

$$P(\omega) = \hbar\omega \frac{4\omega^3}{3\hbar c^3} F(\omega)$$

- ω varies slowly so profile is

$$F(\omega) = \sum_{if} \delta(\omega - \omega_{if}) |\langle f|\mathbf{d}|i\rangle|^2 \rho_i .$$

- Fourier transform is

$$\begin{aligned} \Phi(s) &= \int_{-\infty}^{\infty} e^{-i\omega s} F(\omega)\, d\omega \\ &= \sum_{if} e^{-i\omega_{if} s} |\langle f|\mathbf{d}|i\rangle|^2 \rho_i \end{aligned}$$

with $\Phi(-s) = \Phi(s)^*$, the autocorrelation function.

- monochromatic wave $\implies$ constant Φ
- $\Phi = 0$ implies system at time $(t+s)$ completely uncorrelated with that at time t.
- direct connection between time taken for autocorrelation function to reach zero and the line broadening.

- duration of a collision important.
- $\Phi(s)$ goes to zero shorter than collison time motion of perturbers is irrelevant $\Longrightarrow$ quasistatic approximation
- short times imply large frequency differences so applies to line wings
- collision time is short compared to time needed for $\Phi(s)$ to go to zero $\Longrightarrow$ impact approximation
- quasistatic for protons etc, impact for electrons
- total broadening convolution of two profiles

$$F(\omega) = \int_0^\infty P(\varepsilon_i) I(\omega, \varepsilon_i)\, d\varepsilon_i$$

- ions produce electric field distribution $P(\varepsilon_i)$ (e.g. Holtsmark) electron profile is $I(\omega, \varepsilon_i)$
- must average over field configurations

Convolution of ion field and electron profile

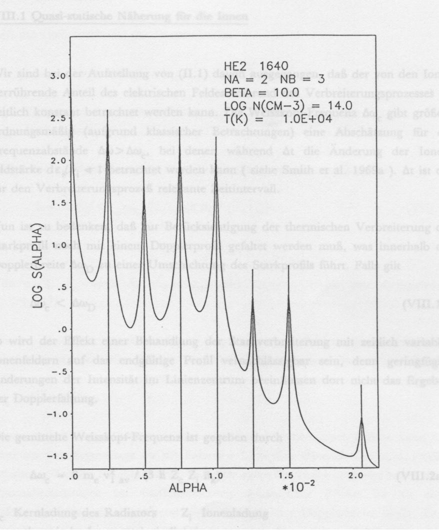


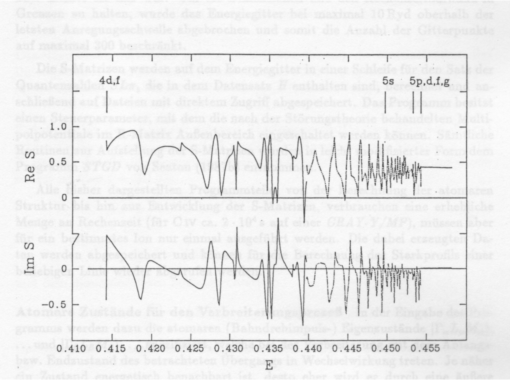
Figure: Schoening, 1988. ALPHA is the normal field strength.

The Impact Approximation

- Baranger showed line profile proportional to

$$(1 - S_i S_f^*)$$

- S is the scattering matrix, is complex
- the line has a shift and a width
- use collision theory to get S gives line profile e.g. Seaton, 1987 uses R-matrix theory (see figure)



Quadratic Stark effect

- ionic contribution negligible
- Dimitrijevic and Konjevic give the following formula

$$\Delta\nu = 6.64 \times 10^{-6} \left(0.9 - \frac{1.1}{z}\right) \left[\sum \left(\frac{3n^*}{2z}\right)^2 ({n^*}^2 - l^2 - l - 1)\right] \frac{n_e}{\sqrt{T}}$$

- Seaton gives

$$\Delta\nu = 6.6 \times 10^{-6} \left(1.3 - \frac{1.2}{z}\right) \left[\sum R^2\right] \frac{n_e}{\sqrt{T}}$$

- R^2 are dipole integrals - from f-values
- many calculations from Sahal-Bréchot and co-workers

Hydrogenic ions

- Griem “Standard Theory”
- Vidal, Cooper Smith “Unified Theory”. Ions static. No impact approximation for electrons.
- Ion dynamic effects?
- MMM - soluble model
- Gigosos et al, Talin et al, Monte Carlo method - are best but limited to low *n*
- Stehle and Hutcheon - extensive tables - not good at very high densities

An example

H_α from Stehle and Hutcheon (1999) $N_e = 3.164 \times 10^{14}$, $T = 10000\,K$