Radiative transfer in stellar atmospheres

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Outline

1. Introduction – stellar atmosphere
2. Model stellar atmosphere
3. Radiative transfer equation
4. Frequency coupling
   (a) within lines
   (b) across lines
   (c) in moving media
5. Boundary conditions
6. Construction of the model stellar atmosphere
7. Formal solution of the RTE

Stellar atmosphere

- part connecting dense stellar core and transparent interstellar medium
- “boundary layer” (Morel, this workshop)
- the only part of the star we directly see
- light carries the only information about astronomical objects
- light influences the state of the stellar atmosphere
  - change of ionization stages
  - change of the population numbers
  - energy transfer $\Rightarrow$ heating
  - momentum transfer $\Rightarrow$ stellar wind

Model stellar atmosphere

standard task of stellar atmosphere physics:

- determination of space distribution of basic physical quantities – $T(\vec{r})$, $n_e(\vec{r})$, $\rho(\vec{r})$, $\vec{v}(\vec{r})$, $J_\nu(\vec{r})$, $n_i(\vec{r})$, ...
- by solving equations
  - energy equilibrium (T)
  - radiative transfer (J_ν)
  - statistical equilibrium (n_i)
  - state equation (n_e)
  - continuity (ρ)
  - motion ($\vec{v}$)

Model stellar atmosphere

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- by solving equations for static case
  - radiative equilibrium (T)
  - radiative transfer (J_ν)
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  - state equation (n_e)
  - hydrostatic equilibrium (ρ)

Model stellar atmosphere

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- by solving equations for static case
  - radiative equilibrium (T)
  - radiative transfer (J_ν)
  - statistical equilibrium (n_i)
  - state equation (n_e)
  - hydrostatic equilibrium (ρ)
- huge system of equations, approximations necessary
- once the atmospheric structure is known, detailed $I_{\mu\nu}$ can be calculated

Model stellar atmosphere

final goal – comparison with observations

Model stellar atmosphere

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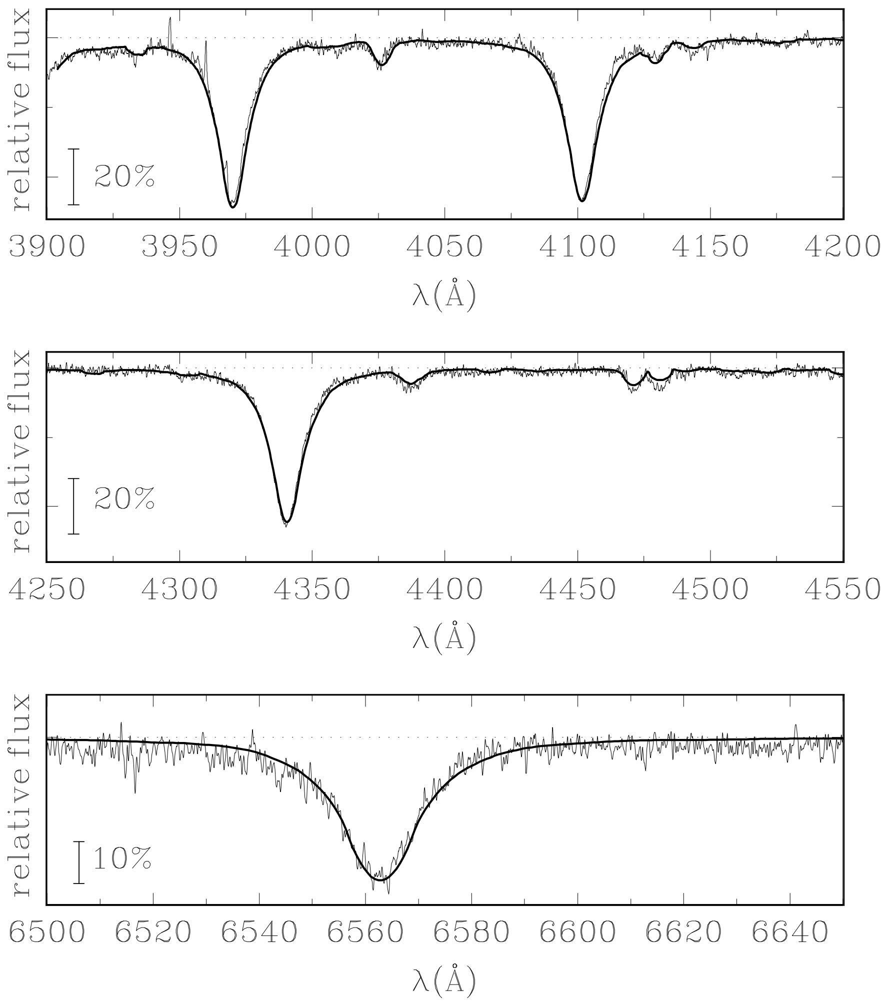

Koubský et al. (1997, A&A 328, 551)

Model stellar atmosphere

final goal – comparison with observations

- usually performed in 2 steps
  1. model atmosphere calculation (structure)
  2. calculation of detailed synthetic spectrum (solution of the radiative transfer equation for a given source function)

  in all steps solution of the radiative transfer equation

Model stellar atmosphere

final goal – comparison with observations

- sometimes performed in 3 steps
  1. model atmosphere calculation (structure)
  2. NLTE problem for trace elements – determination of some n_i for given atmospheric structure
  3. calculation of detailed synthetic spectrum (solution of the radiative transfer equation for a given source function)

  in all steps solution of the radiative transfer equation

Radiative transfer equation

$$\frac{1}{c}\frac{\partial I(\vec{r},\vec{n},\nu,t)}{\partial t} + (\vec{n}\cdot\nabla)I(\vec{r},\vec{n},\nu,t) =$$
$$\eta(\vec{r},\vec{n},\nu,t) - \chi(\vec{r},\vec{n},\nu,t)I(\vec{r},\vec{n},\nu,t)$$

$\eta(\vec{r},\vec{n},\nu,t)$ – emissivity
$\chi(\vec{r},\vec{n},\nu,t)$ – absorption coefficient (opacity)

stellar atmospheres – full spatial 3D desirable

Radiative transfer equation

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simplifying assumptions:

stationarity ($\partial/\partial t \rightarrow 0$)

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static atmosphere ($\vec{v} = 0$)

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3D $\longrightarrow$ 1D plane-parallel:

Radiative transfer equation

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3D $\longrightarrow$ 1D plane-parallel:

$$\mu \frac{\mathrm{d}I(\mu, \nu, z)}{\mathrm{d}z} = \eta(\nu, z) - \chi(\nu, z) I(\mu, \nu, z)$$

$\mu = \cos\theta$ – angle cosine of the ray

Radiative transfer equation

$$\mu \frac{\mathrm{d}I(\mu,\nu,z)}{\mathrm{d}z} = \eta(\nu,z) - \chi(\nu,z) I(\mu,\nu,z)$$

$I(\nu) \longrightarrow I_\nu$:

$$\mu \frac{\mathrm{d}I_{\mu\nu}(z)}{\mathrm{d}z} = \eta_\nu(z) - \chi_\nu(z) I_{\mu\nu}(z)$$

Radiative transfer equation

plane-parallel approximation

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Radiative transfer equation

plane-parallel approximation

$$\mu \frac{\mathrm{d}I_{\mu\nu}(z)}{\mathrm{d}z} = \eta_\nu(z) - \chi_\nu(z) I_{\mu\nu}(z)$$

spherically symmetric approximation

$$\mu \frac{\partial I_{\mu\nu}(r)}{\partial r} + \frac{1-\mu^2}{r} \frac{\partial I_{\mu\nu}(r)}{\partial \mu} = \eta_\nu(r) - \chi_\nu(r) I_{\mu\nu}(r)$$

Coupling in radiative transfer

- spatial – long distance interaction
- angular – scattering
- frequency

crucial in stellar atmospheres

Frequency coupling

- no interaction with matter $\Rightarrow$ no frequency coupling (I constant)

Frequency coupling

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- photon-matter (atoms, molecules, grains) interaction
  - coherent scattering (ν'=ν)
    photons may change direction, but not frequency

    pure scattering RTE

$$\mu \frac{\mathrm{d}I_\mu}{\mathrm{d}\tau} = I_\mu - \frac{1}{2}\int_{-1}^{1} \sigma_{\mu\mu'} I_{\mu'}\,\mathrm{d}\mu' \tag{2}$$

a siginficant part of the Chandrasekhar's Radiative Transfer book

Frequency coupling

- photon-matter (atoms, molecules, grains) interaction
  - line profiles
    natural (Lorentz) profile

$$\varphi_\nu = \frac{\frac{\Gamma}{4\pi^2}}{(\nu - \nu_0)^2 + \left(\frac{\Gamma}{4\pi}\right)^2}$$

$\nu' \neq \nu$
complete or partial redistribution

Frequency coupling

- photon-matter (atoms, molecules, grains) interaction
  - line profiles

Doppler broadening
caused by thermal motions

$$\varphi_\nu = \frac{1}{\Delta\nu_D\sqrt{\pi}} \exp\left(\frac{\nu - \nu_0}{\Delta\nu_D}\right)$$

$$\Delta\nu_D = \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}$$

significant (dominant) for stellar atmospheres

Frequency coupling

coupling of radiation within lines

- complete and partial redistribution
- thermal motions
- Stark broadening, collisional broadening

two-level atom

$$\mu \frac{\mathrm{d}I_\nu}{\mathrm{d}z} = \frac{h\nu}{4\pi} \left[-n_l \varphi_\nu B_{lu} I_\nu + n_u \psi_\nu \left(A_{ul} + B_{ul} I_\nu \right) \right]$$

$\psi(\nu) = \int r\,(\nu', \nu)\,\varphi(\nu')\,\mathrm{d}\nu'$

all line frequencies have to be solved together

Frequency coupling across lines

absorbed photon may be

- reemitted in the same line (transition)
- emitted in a different line or lines (transitions)
- destroyed by collisional transition (heating)

photon may be emitted after collisional excitation (cooling)

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absorption or emission

↓

change of the atomic excitation states distribution

↓

opacity change

Frequency coupling across lines

absorption or emission

↓

change of the atomic excitation states distribution

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opacity change

$$n_i \sum_l (R_{il} + C_{il}) + \sum_l n_l (R_{li} + C_{li}) = 0$$

equations of statistical equilibrium

Frequency coupling across lines

absorption or emission

↓

change of the atomic excitation states distribution

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opacity change

$$n_i \sum_l (R_{il} + C_{il}) + \sum_l n_l (R_{li} + C_{li}) = 0$$

equations of statistical equilibrium

depend on radiation field

Frequency coupling across lines

$$\mu \frac{\mathrm{d}I_{\mu\nu}(z)}{\mathrm{d}z} = \eta_\nu(z) - \chi_\nu(z) I_{\mu\nu}(z)$$

$$\sum_{l \neq i} \{ n_l \left[R_{li} + C_{li} \right] - n_i \left(R_{il} + C_{il} \right) \} = 0, \qquad i = 1, \ldots \mathrm{NL}$$

Frequency coupling across lines

$$\mu \frac{\mathrm{d}I_{\mu\nu}(z)}{\mathrm{d}z} = \eta_\nu(z) - \chi_\nu(z) I_{\mu\nu}(z)$$

$$\sum_{l \neq i} \{n_l \left[R_{li}(J_\nu) + C_{li}\right] - n_i \left(R_{il}(J_\nu) + C_{il}\right)\} = 0, \qquad i = 1, \ldots \mathrm{NL}$$

radiative rates

$$n_i R_{il} = n_i 4\pi \int \frac{\alpha_{il}(\nu)}{h\nu} J_\nu \, \mathrm{d}\nu$$

$$n_l R_{li} = n_l \frac{g_i}{g_j} 4\pi \int \frac{\alpha_{ij}(\nu)}{h\nu} \left(\frac{2h\nu^3}{c^2} + J_\nu\right) \mathrm{d}\nu$$

Frequency coupling across lines

energy conservation

- uneven distribution of spectral lines
- huge amount of lines in the UV region
- large radiative flux in the UV region
- radiation is absorbed in UV, emitted in visual and IR

Frequency coupling across lines

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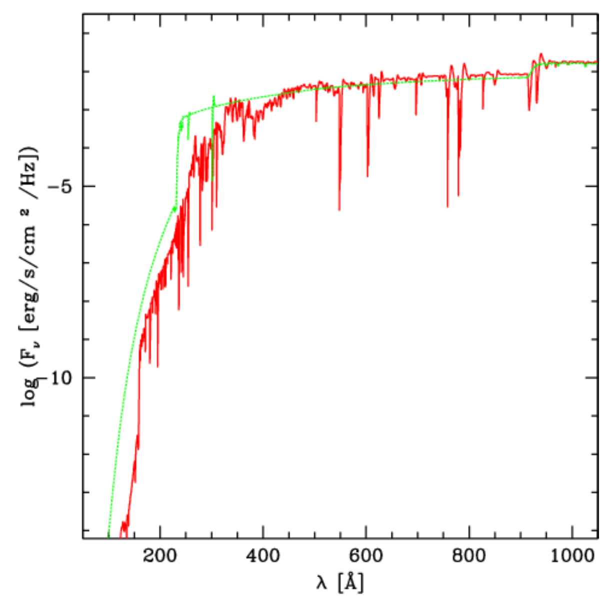
line blanketing

Frequency coupling across lines

line blanketing

- caused mostly by Fe and Ni lines
- Fe II: $\sim 10^7$ lines (Johansson, Kurucz)
- large absorption of high energetic radiation in UV



Martins et al. (2002, A&A 382, 999)

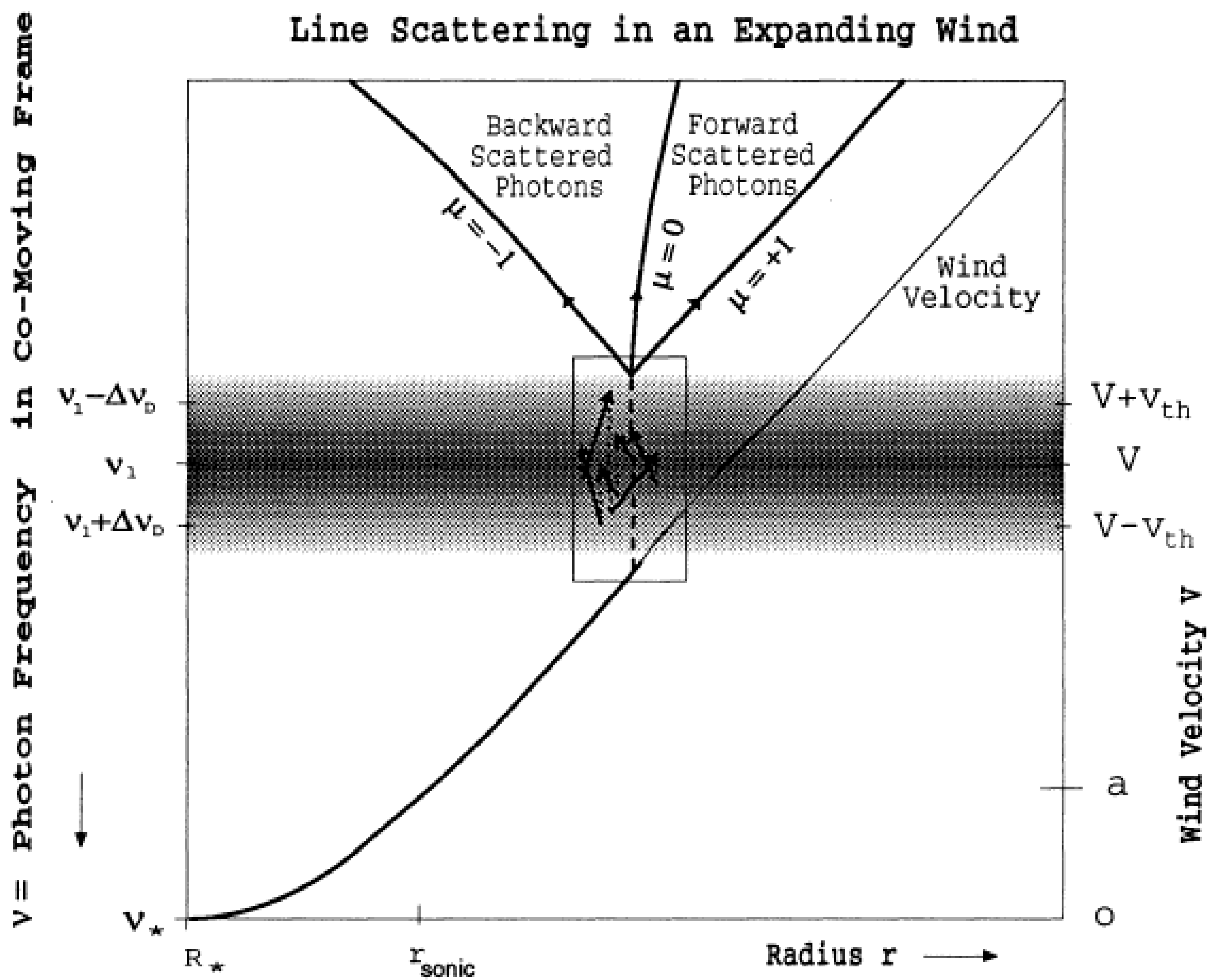
- reemission in other parts of the spectrum

Frequency coupling in moving media

expanding atmospheres ($\vec{v}(\vec{r}) \neq 0$, $\mathrm{d}v/\mathrm{d}r > 0$)

- atomic absorption and emission (scattering) takes place in a rest frame of an atom (comoving frame)
- Doppler shift changes the frequency of lines in a observer frame
- in the observer frame $\nu' \neq \nu$

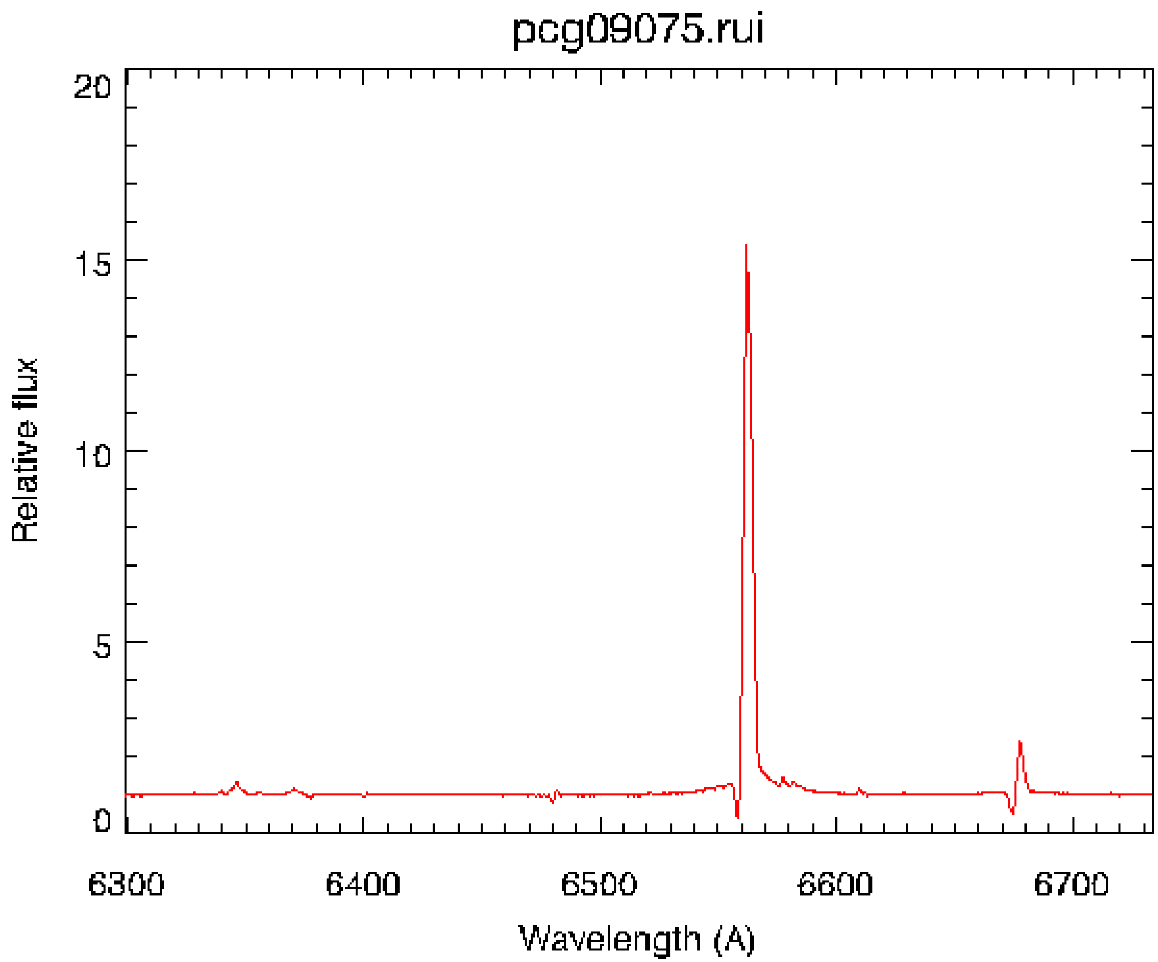
Frequency coupling in moving media



Gayley & Owocki (1994, ApJ 434, 684)

Frequency coupling in moving media

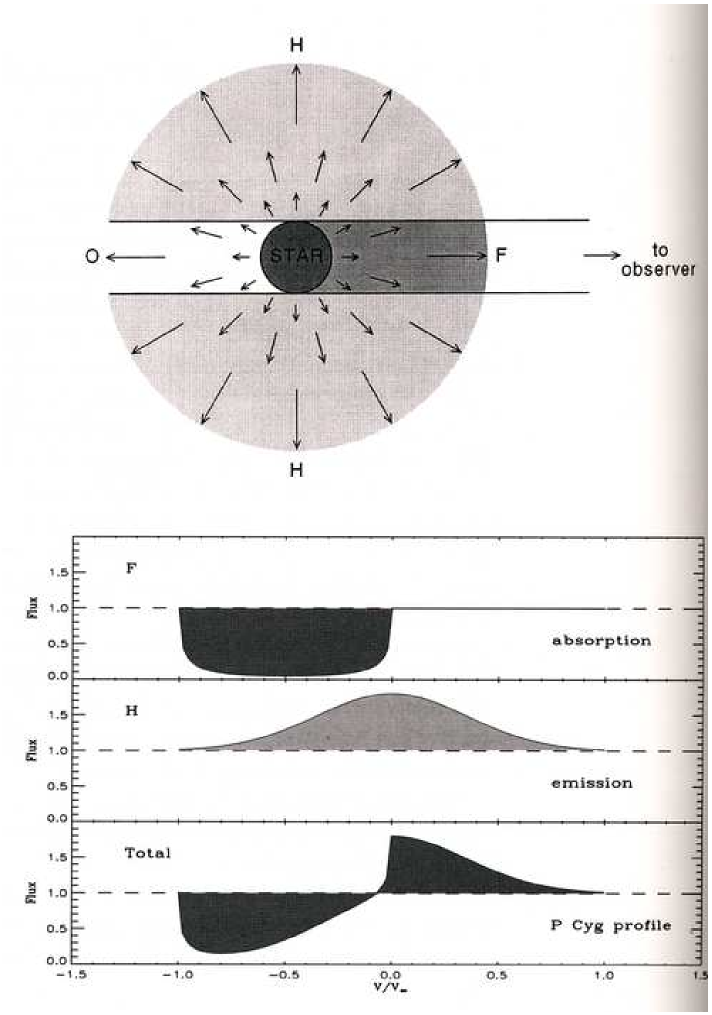
P-Cygni profiles



Ondřejov 29.8.1995

Frequency coupling in moving media

P-Cygni profile formation



Lamers & Cassinelli (1999, Introduction to stellar winds, Cambridge Univ. Press)

Frequency coupling in moving media

radiative transfer equation

$$\mu \frac{\mathrm{d}I_{\mu\nu}(z)}{\mathrm{d}z} = \eta_{\mu\nu}(z) - \chi_{\mu\nu}(z) I_{\mu\nu}(z)$$

$\chi_{\mu\nu}(z)$ and $\eta_{\mu\nu}(z)$ angle dependent

$$\nu' = \nu \left(1 - \frac{\vec{n} \cdot \vec{v}}{c}\right) = \nu \left(1 - \mu \frac{v}{c}\right)$$

Frequency coupling in moving media

comoving frame radiative transfer equation ($\mu' \rightarrow \mu$, $\nu' \rightarrow \nu$)

$$\mu \frac{\partial I_{\mu\nu}(z)}{\partial z} - \left[\frac{\mu^2 \nu}{c} \frac{\partial v}{\partial z} \right] \frac{\partial I_{\mu\nu}(z)}{\partial \nu} = \eta_\nu(z) - \chi_\nu(z) I_{\mu\nu}(z)$$

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static equation

Frequency coupling in moving media

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$$\cancel{\mu \frac{\partial I_{\mu\nu}(z)}{\partial z}} - \left[\frac{\mu^2 \nu}{c} \frac{\partial v}{\partial z} \right] \frac{\partial I_{\mu\nu}(z)}{\partial \nu} = \eta_\nu(z) - \chi_\nu(z) I_{\mu\nu}(z)$$

large velocity gradients – Sobolev approximation

Boundary conditions

stellar atmosphere

- transition from dense stellar core to transparent interstellar medium

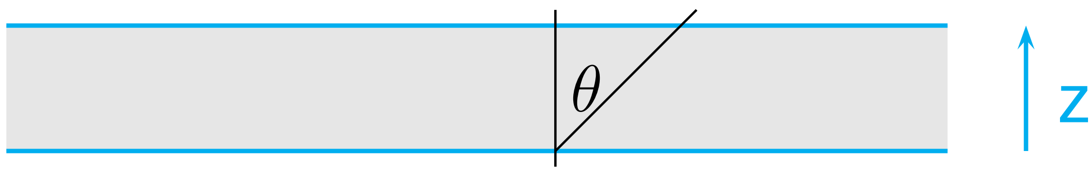
upper boundary condition: $I^- = 0$ (no irradiation)

lower boundary condition: diffusion approximation

$$I_{\mu\nu} = \sum_{n=0}^{\infty} \mu^n \frac{\mathrm{d}^n B_\nu}{\mathrm{d}\tau_\nu^n} = B_\nu + \mu \frac{\mathrm{d}B_\nu}{\mathrm{d}\tau_\nu} + \mu^2 \frac{\mathrm{d}^2 B_\nu}{\mathrm{d}\tau_\nu^2} + \cdots$$

Model atmosphere construction

simplified situation: 1D static plane-parallel atmosphere



we take into account

- radiation from all directions (one angle variable)
- full frequency spectrum

Model atmosphere construction

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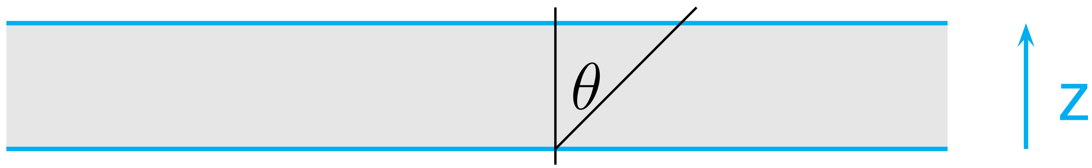

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solution of

- radiative transfer equation ($I_{\mu\nu}$)
- equation of radiative equilibrium (T)
- equation of hydrostatic equilibrium (ρ)
- equations of statistical equilibrium (n_i)

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- equation of radiative equilibrium (T)

$$4\pi \int_0^\infty (\chi_\nu J_\nu - \eta_\nu)\ \mathrm{d}\nu = 0$$

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$$\frac{\mathrm{d}p}{\mathrm{d}m} = g - \frac{4\pi}{c} \int_0^\infty \frac{\chi_\nu}{\rho} H_\nu \, \mathrm{d}\nu$$

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- equation of radiative equilibrium (T) T given

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- equation of hydrostatic equilibrium (ρ) ρ given

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$$n_i \sum_l (R_{il} + C_{il}) + \sum_l n_l (R_{li} + C_{li}) = 0$$

Formal solution of the RTE

next step after calculation the model atmosphere and the occupation numbers

any solution for given χ_ν and η_ν (given S_ν) – relatively simple

- 1st order differential or integral equation
- 2nd order differential equation – common in stellar atmospheres (Feautrier solution)

crucial for the total accuracy of the whole problem solution

Feautrier solution of the RTE

solution along a ray (specific intensities I^+ and I^-)
introduce “Feautrier variables”

$$u_{\mu\nu} = \frac{1}{2}\left(I^+_{\mu\nu} + I^-_{\mu\nu}\right)$$
$$v_{\mu\nu} = \frac{1}{2}\left(I^-_{\mu\nu} + I^-_{\mu\nu}\right)$$

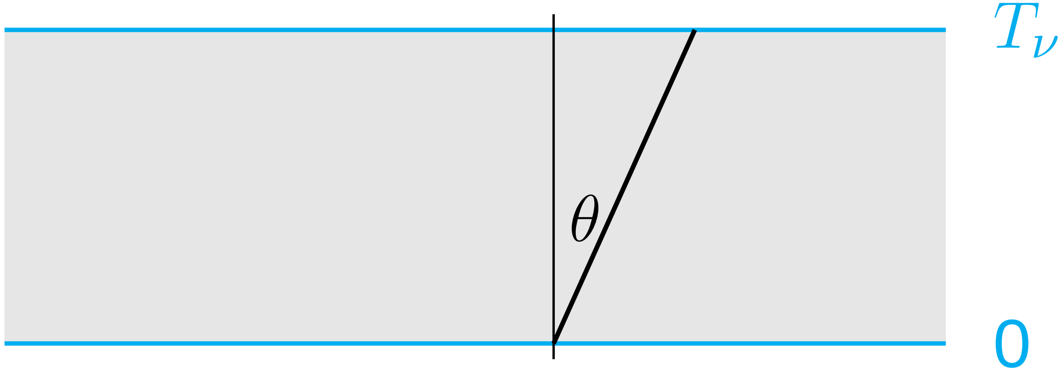
the transfer equation

$$\frac{\mathrm{d}^2 u_{\mu\nu}}{\mathrm{d}\tau^2_{\mu\nu}} = u_{\mu\nu} - S_\nu$$

$+$ boundary conditions

Short characteristics

for a finite slab



outward direction

$$I^+_{\mu\nu}(\tau_\nu) = I^+_{\mu\nu}(T_\nu)\, e^{-\frac{T_\nu - \tau_\nu}{\mu}} + \frac{1}{\mu}\int_{\tau_\nu}^{T_\nu} S(t)\, e^{-\frac{t-\tau_\nu}{\mu}}\,\mathrm{d}t$$

inward direction

$$I^-_{\mu\nu}(\tau_\nu) = I^-_{\mu\nu}(0)\, e^{-\frac{\tau_\nu}{\mu}} + \frac{1}{(-\mu)}\int_0^{\tau_\nu} S(t)\, e^{-\frac{\tau_\nu - t}{\mu}}\,\mathrm{d}t$$

Conclusions

radiation in stellar atmospheres

- carries the only information about astronomical objects
- influences the state of the stellar atmosphere
  - change of ionization stages
  - change of the population numbers
  - energy transfer $\Rightarrow$ heating
  - momentum transfer $\Rightarrow$ stellar wind
- spatial, angle, and frequency coupling